

# WORKSHOP ON DYNAMICAL SYSTEMS AND APPLICATIONS



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Hotel PIRAMIDA, Maribor, Slovenia  
June 27 - 28, 2025

## PROGRAMME

# WORKSHOP ON DYNAMICAL SYSTEMS AND APPLICATIONS

Hotel PIRAMIDA, Maribor, Slovenia  
27 June 2025 - 28 June 2025

## Organizers

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Prof. Dr. Brigita Ferčec (co-chair)  
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Prof. Dr. Xingwu Chen  
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Prof. Dr. Yilei Tang  
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## SCHEDULE

Friday 27 June	
13:00-14:30	Lunch (Hotel Piramida)
14:30-15:00	Registration and Opening
Chairman	<b>Romanovski</b>
15:00-15:40	<b>X. Zhang</b>
15:40-16:20	<b>Chen</b>
16:20-16:40	Tea & Coffee
16:40-17:20	<b>Sterpu</b>
17:20-18:00	<b>Mencinger</b>
19:00	Welcome dinner (Novi Svet)

Saturday 28 June	
Chairman	<b>Mencinger</b>
10:00-10:40	<b>Li</b>
10:40-11:20	<b>Mincheva</b>
11:20-11:40	Tea & Coffee
11:40-12:20	<b>Efrem</b>
12:20-13:00	<b>Wang</b>
13:00-15:30	Lunch
Chairman	<b>Ferčec</b>
15:30-16:10	<b>Romanovski</b>
16:10-16:50	<b>W. Zhang</b>
16:50-17:10	Tea & Coffee
17:10-17:50	<b>Xiao</b>
17:50-18:30	<b>Tang</b>
19:30	Conference dinner (Ancora)

# Adresses of All Participants

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# Abstracts

# Limit cycles of piecewise-linear differential systems

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We talk about the dynamics of piecewise-linear differential systems including the numbers of crossing limit cycles, sliding limit cycles as well as their location, and bifurcations about singularities in cases of dimension 2 and dimension 3.

## References and Literature for Further Reading

- [1] R. Cristiano, D. J. Pagano, Two-parameter Boundary Equilibrium bifurcations in 3D-Filippov systems, *J. Nonlinear Sci.* **29** (2019) 2845-2875.
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# Local Dynamics in a Generalized Lotka–Volterra Model with Infinitesimal Parameters

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This study focuses on a two-dimensional cubic Lotka–Volterra model characterized by the presence of infinitesimal parameters. The analysis is centered on two specific degenerate cases. In each degenerate case, the local dynamics near equilibrium points is thoroughly investigated. To capture the full complexity of the system’s behavior, a detailed bifurcation analysis is performed. This results in sixteen distinct bifurcation diagrams—graphical representations that map out how the qualitative nature of the system’s trajectories changes as parameters vary. These diagrams are partitioned into a total of forty regions, each corresponding to a qualitatively different dynamical regime. Together, these bifurcation diagrams provide a comprehensive classification of the system’s possible behaviors under the two degenerate cases, offering valuable insight into the rich dynamics that can arise even in models such as Lotka–Volterra systems.

## References and Literature for Further Reading

- [1] G. Moza, D. Constantinescu, R. Efrem, An analysis of a class of Lotka-Volterra systems, *Qualitative Theory of Dynamical Systems* **21**, Article number: 32 (2022).
- [2] R. Efrem, M. Sterpu, Local dynamics and bifurcation for a two-dimensional cubic Lotka-Volterra system (I), *Annals of the University of Craiova-Mathematics and Computer Science Series* **50** (2023), no. 1, 247-258.

# Complexity of orbits of points under Lüroth map

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Lüroth map  $T : [0, 1) \rightarrow [0, 1)$  is given by

$$T(x) = \begin{cases} n(n+1)x - n, & \text{if } x \in [\frac{1}{n+1}, \frac{1}{n}) \\ 0, & \text{if } x = 0. \end{cases}$$

Each irrational number  $\bar{x} \in (0, 1)$  corresponds to a unique sequence  $x = \{x_i\} \in \mathbf{N}^{\mathbf{N}}$  in the sense that

$$x_i = n \text{ if and only if } T^{i-1}(\bar{x}) \in \left[ \frac{1}{n+1}, \frac{1}{n} \right)$$

where  $T^i$  is the  $i$ -th iteration of  $T$ . Then for each  $n$ , one can get an infinite probability vector  $\Pi(x|n) = (p_{i,n})_{i \in \mathbf{N}}$  where  $p_{i,n}$  is the frequency of  $i$  occurring in the prefix of  $\{x_i\}$  of length  $n$ . Let  $A(\{\Pi(x|n)\}_{n \in \mathbf{N}})$  be the set of accumulation points of the sequence  $\{\Pi(x|n)\}_{n \in \mathbf{N}}$ . Given a set  $C$ , let

$$\Omega_{=C} = \{x \in \mathbf{N}^{\mathbf{N}} : A(\{\Pi(x|n)\}_{n \in \mathbf{N}}) = C\}$$

and

$$\Omega_{\subseteq C} = \{x \in \mathbf{N}^{\mathbf{N}} : A(\{\Pi(x|n)\}_{n \in \mathbf{N}}) \subseteq C\}.$$

In this talk, we present a way to analyse the complexity of  $\pi(\Omega_{=C})$  and  $\pi(\Omega_{\subseteq C})$ . This is a joint work with Y.X. Gui and Y. Zhou.

## References and Literature for Further Reading

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# On Stability of the Homogeneous Planar Quadratic Systems

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In this presentation, we consider the stability of homogeneous planar quadratic systems of ordinary differential equations by associating each vector field with a commutative, non-associative algebra, we translate dynamical stability into algebraic invariants. First, we prove that such system admits a stable origin exactly when its vector field factorizes linearly, corresponding to the existence of a single nilpotent line and the absence of nontrivial real idempotents in the corresponding algebra. Next, by considering the complexification, we establish that stability further requires the corresponding complex algebra to be spanned by two nonzero (complex) idempotents, while the real algebra remains trivial. These algebraic criteria provide a complete classification of planar systems with stable singularities and underscore the pivotal role of idempotent structure in governing invariant flows. We also outline how those results offer a promising path for higher-dimensional generalizations.

## References and Literature for Further Reading

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# Dynamics of Mass Action Kinetics Systems

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Mass action models of reaction networks are often represented by parametric non-linear ODE systems. Multistationarity and oscillations are ubiquitous phenomena in biochemical reaction networks models. We present a method for finding regions in parameter space for multistationarity. For oscillations, Hopf bifurcation points are identified. The mathematical methods are based on degree theory, algebraic geometry and convex analysis. Some examples from biochemical reaction networks will be presented.

# Invariants and reversibility in polynomial systems of ODEs

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We present the results of the paper [1] which explores a relationship between invariants of certain group actions and the time-reversibility of two-dimensional polynomial differential systems exhibiting a  $1 : -1$  resonant singularity at the origin. We focus on the connection of time-reversibility with the Sibirsky subvariety of the center (integrability) variety [2], which encompasses systems possessing a local analytic first integral near the origin. An algorithm for generating the Sibirsky ideal for these systems is proposed and the algebraic properties of the ideal are examined.

Furthermore, using a generalization of the concept of time-reversibility [4, 3] we study  $n$ -dimensional systems with a  $1 : \zeta : \zeta^2 : \dots : \zeta^{n-1}$  resonant singularity at the origin, where  $n$  is prime and  $\zeta$  is a primitive  $n$ -th root of unity. We study the invariants of a Lie group action on the parameter space of the system, leveraging the theory of binomial ideals as a fundamental tool for the analysis. Our study reveals intriguing connections between generalized reversibility, invariants, and binomial ideals, shedding light on their complex interrelations.

## References and Literature for Further Reading

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- [4] S. Walcher, On transformations into normal form, *J. Math. Anal. Appl.*, **180** (1993), 617–632.

# Bifurcation Diagrams for a Cubic Kolmogorov System

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A two-dimensional cubic Kolmogorov system depending on two small parameters is considered. The study investigates the local dynamics and bifurcations in a neighbourhood of the origin of the phase plane, when the parameters are sufficiently small. The analysis assumes some additional hypotheses on the coefficients of the second degree terms are satisfied, thus it complements a previous work. Compared to the later, the setting in this study seem to give the "difficult case", as new bifurcations (such as Hopf or homoclinic bifurcations) are found. The complex dynamics of the system is illustrated by a comprehensive classification with respect to some of the coefficients and by detailed bifurcation diagrams for each of the identified case.

## References and Literature for Further Reading

- [1] G. Moza, D. Constantinescu, R. Efrem, An analysis of a class of Lotka-Volterra systems, *Qualitative Theory of Dynamical Systems* **21**, Article number: 32 (2022).
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# Limit cycles for glycolytic differential systems

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The aim of this talk is to investigate the problem of limit cycles and global dynamics for the general case of the Selkov system. By applying limit cycle theory for Liénard systems, we first transform the Selkov systems into Liénard systems. Then the theory and techniques related to limit cycles for Liénard systems can be utilized and further developed. Additionally, we obtain a new criterion for the uniqueness of limit cycles in general Liénard systems. Consequently, we address the majority of parameter cases for the conjecture regarding the uniqueness of limit cycles proposed in the literature. Finally, we present the global bifurcations and dynamical structures of the system in the Poincaré disc.

## References and Literature for Further Reading

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# Global dynamics of several non-smooth oscillators

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The aim is to study several non-smooth Liénard systems. By qualitative and bifurcation analysis, the oscillator contains abundant nonlinear phenomena, including the double limit cycles bifurcation, figure-eight loop bifurcation, heteroclinic bifurcation, coexistence of a crossing limit cycle and a pseudo homoclinic loop, coexistence of two crossing limit cycles and the existence of pseudo saddle-node loops.

# From local to global dynamics in Kolmogorov polynomial vector fields

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In this talk we will introduce an approach to characterize global dynamics from local linearized dynamics of Kolmogorov polynomial vector fields, and establish a link between the integrability of the vector field and the intersection number of the corresponding algebraic curves. Specially, a new criterion on nonexistence of limit cycles is given for Kolmogorov polynomial vector fields with any degree  $n$ . Last we give two examples to show how to use the criterion. This is a joint work with Dr. Hongjin He.

# Qualitative Analysis of Zero Distribution and Application to Enzyme-Catalyzed Reaction Model

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In this talk we discuss a substrate-activator system, which depends on a cubic polynomial with such a complicated relation between its coefficients and the original parameters that the coordinates of equilibria or even the number of equilibria can hardly be determined in many cases. All found results on its qualitative properties and bifurcations are given indirectly for the artificial parameter  $s_*$ , a coordinate of a general equilibrium, and the analysis of its dynamics remains far from completion. Not following the common idea of computing eigenvalues at equilibria, we give a complete analysis of equilibria directly for those original parameters by using continuity, monotonicity and some techniques of inequality. For a global analysis we discuss its equilibria at infinity, one of which possesses degeneracy so high sometimes that neither the wellknown normal sector method nor the blowing-up method can be used easily. Furthermore, overcoming those difficulties from not solving all coordinates of equilibria, we give a versal unfolding with its original parameters to the degenerate cases and present bifurcation curves of periodic orbits and homoclinic orbits explicitly. This is a joint work with Yilei Tang and Deqing Huang.

## References and Literature for Further Reading

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# Reduction of Elementary Integrability of Polynomial Vector Fields

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Prelle and Singer showed in 1983 that if a system of ordinary differential equations defined on a differential field  $K$  has a first integral in an elementary field extension  $L$  of  $K$ , then it must have a first integral consisting of algebraic elements over  $K$  via their constant powers and logarithms. Based on this result they further proved that an elementary integrable planar polynomial differential system has an integrating factor which is a fractional power of a rational function. Here we extend their results and prove that any  $n$  dimensional elementary integrable polynomial vector field has  $n - 1$  functionally independent first integrals being composed of algebraic elements over  $K$ . Furthermore, using the Galois theory we prove that the vector field has a rational Jacobian multiplier. This talk is based on the papers in the references.

## References and Literature for Further Reading

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